Evaluation of the Research and Professional Activity of the Institutes of the Czech Academy of Sciences (CAS) for the period 2010–2014

Final Report on the Evaluation of the Institute

Name of the Institute: Institute of Mathematics of the CAS, v. v. i.

Fields, in which the Institute registered 5 teams:

Mathematics

Observer representing the Academy Council of the CAS: Michal Haindl
Observer representing the Institute: Pavel Krejčí,
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Commission No. 1: Mathematics
Chair: Professor Willi Jäger

Date of the visit of the Institute: November 30
Program of the visit of the Institute: see attached Minutes from the visit

Evaluated research teams:
No. 1 Algebra, Geometry and Mathematical Physics
No. 2 Differential Equations and Theory of Integral
No. 3 Evolution Differential Equations
No. 4 Numerical Analysis
No. 6 Topology and Functional Analysis

A.   Evaluation of the Institute as a whole

Remark:
Several of the statements, the Commission has to make, apply also to other Institutes and the teams, which had to be evaluated. The Commission decided to formulate them more in detail and to send them in a letter to the President of CAS and the Chair of the Coordination Board of the Evaluation. We are going to refer to them by S and their number as listed in the letter.

1. Introduction

The Institute of Mathematics comprises 6 departments having their scientific focus in basic mathematical research, but also strongly linked to important application areas like general relativity, quantum mechanics, material sciences, fluid dynamics, solid mechanics, computational sciences and computer sciences. Its founder generation and the second generation included top scientists like Eduard Čech (topology, differential geometry), Ivo Babuška (numerical analysis), Miroslav Fiedler (linear algebra, matrix theory, graph theory), Jaroslav Kurzweil (ordinary differential equations, theory of integral), Vlastimil Pták (functional analysis, numerical analysis, linear alge-

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bra) and Jindřich Nečas (partial differential equations, functional analysis, fluid mechanics), Zdeněk Frolík (functional analysis, topology) and Petr Hájek (logic). Also now, several of members of the Institute are top leaders in their fields. They all have been setting the quality standards and directly or indirectly determined the direction of future scientific development not just for the Institute. This fact might seem to be "orthogonal" to present rules of science management, following business methods and relying strongly on quantitative indicators to evaluate scientific quality. The Committee Mathematics is glad not to have come into conflicts with this fact when evaluating this Institute. The necessary condition that the Institute has several top scientists, leading their teams, is still fulfilled. However, the Commission emphasizes strongly that all responsible authorities should do their utmost to maintain or, even better, to improve the conditions that will allow attracting the best scientists to leading positions in the Institutes of CAS.

2. Strengths and Opportunities
The Institute is scientifically in an excellent and favourable condition. It was able to continue and to further develop those mathematical disciplines for which Czech science has been famous, and to open its research to new directions. E.g. a prominent example is provided by the Team 3 Evolution Differential Equations, which is a focal point for research in the mathematical theory of fluids. The team in Logic has extended its scientific program to Theoretical Computer Science, a positive and natural development, as long as the basic problems of Mathematical Logics are not lost. There is a constant fruitful exchange between theory and application oriented research to the benefit of both sides. The Institute is opening up to new application areas like Life Science, a move, which in principle is to be welcomed. This is, however, limited in its potentials due to the limited resources.
Quality of the research has to have top priority, following the guidelines valid for the Institute.
The prestigious ERC Advanced Grants and the Marie-Curie Action Mobility Grant are indicators for top quality in research.
Most of the teams of the Institute are integrated in international networks of high scientific quality. The Institute as whole is perfectly linked on the European level. Its visitor program is in quality and size convincing and can be considered as a good investment for science. The activities in organizing conferences, workshops and school are internationally well accepted. The series of Equadiff conferences is probably the most important event, alone by its traditions and the effects it had on the international scientific and human exchange in the past.
The Institute is perfectly co-operating with partner teams, in particular at Charles University, and also keeps close contact to CAS departments and teams, with a strong orientation towards Mathematics, that for historical or other reasons are located in other institutes. Combining all in one virtual node, one obtains an impressive scientific potential, which is internationally very competitive.
The Nečas Center for Mathematical Modeling, a joint activity of the Institute and the Faculty of Mathematics and Physics, is following very promising concepts in research and in education. It is providing not only the framework for joint research projects, but has also the potential to establish and host a graduate school, which would be a substantial contribution improve the PhD education. The open key problems are the financial resources.
The Institute is very active in promoting Mathematics and its Applications in almost all areas of human activities.
The scientific and administrative management of the Institute is working very well. The Director and the Board deserve recognition for directing the Institute with great expertise and the vision, necessary for scientific centers.

3. Weaknesses and Threats

Only Team 2 does not meet the high standards of the other teams. The main reasons for this situation are problems in the age structure and the lack of a top scientist in the proper age group to lead and to influence the scientific direction of the team, which is the case for the other teams of the institute. The Institute and the Academy have to take measures to change this situation. Just minor corrections of the research topics will not help. Large deviations of the tracks seem to be required.

Most teams share the problems with the age structure and the difficulties with attracting top senior scientists and highly qualified junior scientists. This fact led to the reduction of core topology in Team 6, which has to be considered very unfortunate.

The main obstacles to including new research areas, coming up in particular at the interfaces of areas, are the lack of financial resources and of highly qualified junior researchers.

The problem to attract bachelor, master and PhD students to the research fields of the Institute led to a broader discussion during the visit. The number of these students is obviously too small, and a change has to be achieved. The Commission is convinced that a joint agreement to adjunct selected members of the Institute to the Faculty of the University can be achieved and this part of the problem solved.

There exist important sub-disciplines of Mathematics, which are not represented in the Institute for Mathematics. Stochastics Analysis and Statistics, Optimization and Control Theory are examples of such disciplines, which are at least partially represented in other Institutes of CAS, and more or less imbedded in application areas. However, these disciplines cannot just stay appendices to mainly applied research, located at the moment in different institutes, but they should also be represented inside the Academy by basic research.

At any rate, it is necessary for the Institute of Mathematics to build up and maintain expertise in these fields, in order to stay competitive, since also in Mathematics exciting innovations happen at the interfaces of disciplines.

Seen from the overall perspective of the Academy, Numerical Mathematics has to be strengthened to meet the growing requirements and the great need for numerical methods. The resources available to Team 4 Numerical Analysis are too limited and allow it to make only a limited contribution.

4. Recommendation

In order to maintain the top scientific quality of the Institute several measures have to taken:

Within the Institute (S2)

1. focusing of the research program of the team to central topics,
2. identifying and selecting new research directions,
3. allocating the existing resources following defined priorities.

According to the results of this evaluation, Team 2 will be mostly affected, since its research program and its structure have to be reconsidered (S3). Here the Director and the Board have to act both sensitively and farsighted. Since a reorientation will be necessary, the Institute and the Academy should use the chance to support a restart but in a new direction.

Furthermore, the staff necessary to represent Numerical Analysis appropriately should be assigned to Team 4 under the requirement that its program is a part of an integrated program in Numerical Mathematics in the CAS. The Institute and its teams should help to promote new application areas, supplying mathematical and computational methods and tools. However establishing a competitive group e.g. in Modeling and Simulation in Biosciences needs a broader

competence, which is usually not available in a team with just expertise in analytical or computational methods.

Between the Institute and Universities (S4, 5, 6):
The Commission is recommending a consultation of the Academy and the Institute on one side and the Universities and the Faculties on the other side with the aim, to improve the cooperation in particular in the education of students on all levels.

The Commission strongly urges the Institute and the Academy to continue their support for the Nečas *Center for Mathematical Modeling* as a platform for joint research projects and educating and guiding joint PhD students.

The Commission realized in this evaluation that reforms in the PhD education in Czech Republic are urgently needed. The arising problems can be solved only at the highest decision-making level. All activities of the Institute and its partners contributing to changes deserve full support.

The *Nečas Center* should be also developed as central node of a national network in Computational Sciences linked to High-Performance-Scientific-Computing and the national Supercomputing Center in Ostrava.

5. Detailed evaluations

Declaration on the quality of the results and share in their acquisition
The Phase I evaluation rated the outputs submitted by all 5 teams, which were assigned to the Commission Mathematics, as follows:

33 % „*world leading*", 44 % „*internationally excellent*", 22 % „*recognized internationally*"

The less favourable result for Team 2 still remains in the same categories, but is shifted closer to "recognized internationally".

This provides a first-hand impression of the quality of the institute´s research.

A more detailed description of these outstanding results, which were also confirmed in the presentations and discussions during the visit, is given in the report on the individual teams.

Declaration on the involvement of students in research
The Institute is very engaged in teaching and prepared to even strengthen the advising of students on all levels, on bachelor, master and PhD level. However, there is a complaint in all teams that despite of close relations to the Universities, the early access to students, necessary to attract the most talented ones, is very difficult. The Committee did not see the formal reasons for this situation, which should be changed to the profit of the Universities and the Academy. The students, who could be attracted, were properly integrated in the research projects, according to their level of training.

Declaration on societal relevance
The Institute is strongly involved in basic research in Mathematics, the impact of which very often cannot be seen directly (S1). The members were engaged in several activities to make the importance of Mathematics for society visible for the public.

The Institute is also active in providing access to mathematic literature and in evaluating scientific publications as a service to the scientific community. For more information see the reports on the teams.

Declaration on the position in the international and national context

The Institute is a landmark in mathematical basic research, representing Czech science prominently.

Plenty of evidence for its distinguished qualities is provided. Two ERC Advanced Grants are the most prestigious acknowledgements. Several of its leading scientists are in positions with great influence on the scientific development of their own field and of mathematics in general.

By tradition the Institute is also strongly oriented to applications. Due to limited resources, only those fields are covered by research teams, where Czech mathematicians have been internationally at the forefront. Since the foundation of the Institute, sciences have changed in particular due to the revolutions in the Information Technology. These changes effected also both theory oriented and also application oriented research in mathematics. The teams in Logic and Numerical Analysis are most affected, however also the other teams, where areas like computational fluid and solid mechanics, computational topology or general relativity based on mathematical concepts and methods have been making significant progress. The Institute of Mathematics as well as CAS are aware of this situation that requires changes without losses for its leading research teams.

Declaration on the vitality and sustainability

The activities in research and education, the very active national and international co-operations show that the scientific teams, the service and the management teams are alive, open and flexible. The sustainability depends strongly on the availability of resources and structures to attract top scientists and the best of the young generation. In order to initiate and to pursue new research directions, additional resources are needed. The development of new strategies was started, however, the discussion is still going on. It should take into account also the advise of the Scientific Advisory Board, looking from outside at the Institute like the current Committee, while being more closely involved with the on-going operation.

Declaration on the strategy and plans for the future

The Institute decided to continue taking excellence as a guideline of its strategy and to support cooperation in centers, integrating expert teams from different institutions. As a consequence of this evaluation the scientific and administrative management must face the task of finding solutions in setting directions of research and weights of support properly. Whereas the Teams 1, 3 and 5 and the Team Logic and Theoretical Computer Sciences will not raise team specific discussions, the future development of Team 2 also of Team 4 has to be further debated in more detail, as already remarked. In this discussion the research program in general and the role of Numeric in particular will have to be discussed.

B. Evaluation of the individual teams

Evaluation of the Team No. 1:

Algebra, Geometry and Mathematical Physics

1. Introduction

This team has as central aim to build bridges between basic research in mathematical disciplines investigating structures important in basic physics, connecting:
differential geometry and general relativity, functional analysis and quantum mechanics,
complex functions and conformal field theory, vertex operator algebras,
algebraic topology, geometry and string field theory, symplectic geometry and mechanics,
topological field theory, group theory and representation and quantum groups.
These research areas are internationally highly topical and have had a high impact at least on the development of mathematical theory.

2. Strengths and Opportunities

The team is despite of its small size playing an important role in the international competition in this hot research fields. It is rather young, very international and competitive. It deserves the acknowledgement it is getting.

3. Weaknesses and Threats

The team is not equipped to the extent necessary to start new topics without cuts in the other very successful research areas. Opening a new topic like Information Geometry in an intensity needed to become and stay competitive requires new resources. Attracting young researchers is also for this team difficult, despite its scientific situation.

4. Recommendations

The team is asked to continue and to strengthen the contacts to the experts in Physics and as far as Information Geometry is concerned, also in Stochastics and Statistics. Some of the research areas also have a computational aspect like mathematics of general relativity. It is recommended to follow also the development of the computational approaches.

6. Detailed evaluations

Declaration on the quality of the results and share in their acquisition
This is a relatively small team that covers two main areas of research:
1. Mathematical relativity, exact solution of the Einstein equations of gravity.
2. Algebraic topology, cohomology and homotopy theory,
   deformation theory of algebras,
   algebraic quantum field theory.
In addition, there is only a very small group working on

3. Information geometry.

The connection between the two main areas is through the use of algebraic and geometric structures. The team is specialized in identifying solutions of the Einstein equations with special symmetries. The information geometry group is so far embryonic, but may develop into a promising area of applications where the team as a whole can bring in its expertise.

In all three areas the team is strong and is internationally well connected and highly recognized. They make significant contributions.

The publication record is excellent, both with respect to quantity and quality. The team publishes mainly in the highest-ranking international journal in their field of specialisation, such as Classical and Quantum, Gravity, Physical Review D, Advances in Mathematics, etc. They also produced two well-published monographs, with a third in the review process at Springer.

18% of the submitted outputs were rated „*world leading*", 64% „*internationally excellent*", 18 % „*recognized internationally*" in Panel I.

Declaration on the involvement of students in research

Regarding the involvement of students, the team share the difficulties encountered by all CAS institutes. 4 master and 4 PhD students are reported for the evaluated period.

Declaration on societal relevance

The teams research contributes in the first to main two research area to better understanding of the basic physical laws and their consequences. They are contributions to human culture and to knowledge, which sooner or later might also lead to technological progress. Getting into Information Theory by applying concepts and methods of Differential Geometry to Probability may open up a more direct access to relevant applications.

The involvement in teaching is relatively low, also the supervision of bachelor and master theses - a fact, which seems to be in contradiction to the involvement in educating the young generation. The team participates regularly in the organisation of a winter school on physics and geometry in Srni as well as in the organisation of conferences, both in the Czech Republic and abroad attracting in particular young scientists.

The team has no cooperation with the business sector. This appears adequate given the research orientation of the team. There have been several activities towards the popularisation of science, notably also through lectures on mathematics for high school students.

Declaration on the position in the international and national context

The team is working in some aspects at the leading edge of research in their fields. It has numerous international collaborations and has attracted international visitors. Members of the team have had extensive visits abroad at distinguished institutions, underlining the relevance of the research at an international level.

The situation concerning funding through grants seems quite satisfactory. However, what applies to all teams and departments: it would be desirable that extra financial resources could be made accessible for the financing of Ph.D. students and postdocs.

Declaration on the vitality and sustainability

The current composition of the team is fairly reasonable. The qualification of the existing team members is excellent. Half of its members are foreigners, which is quite unusual. The team is also comparatively young.

Declaration on the strategy and plans for the future
The proposed research plan for the near future is reasonable. Basically, the team follows a research plan that has been successful so far and is expected to be sustainable for the foreseeable future. This can be achieved with the available resources.
Regarding the area of Information Geometry, it will have to be seen whether this can develop into a more important area of the team. This would probably hinge on the possibility to increase the manpower in this area. If this happens, it appears to be advisable to strengthen the contact to Stochastics and Statistics.

Evaluation of the Team No. 2:

Differential Equations and Theory of Integral

1. Introduction
In the focus of the research of this team is the analysis of solutions of ordinary differential and difference equations, functional- differential equations of mainly of ordinary type, of existence, uniqueness stability, asymptotic behaviour and qualitative properties. Considering functionals in the equations leads is in general to investigation of dynamical systems in infinite dimensional spaces and to the analysis of the properties of their solutions. Integration theory is playing an important role, particular the Kurzweil-Stieltjes integral and its properties, leading to an expanded concept of solutions, opening up larger areas of applications, in theory and in practice.
The Brno branch was founded by Otakar Borůvka, whereas Jaroslav Kurzweil founded the group in Theory of Integral, both top scientists. Kurzweil is despite of his high age still active, but can no longer assume responsibilities in managing the team.

2. Strengths and Opportunities
The most substantial contributions could be achieved for the theory and the application of the Kurzweil-Stieltjes integral and generalized ordinary differential equations. The concepts and results of these investigations are likely to open new perspectives also in Stochastic Analysis, which may be pursued in the future. The achievements in the analysis of oscillations deserve to be specially emphasized.
The involvement of the members in teaching and education is fully appreciated.

3. Weaknesses and Threats
With exception of the mentioned highlights the overall rating is below the standards of Institute of Mathematics, which are admittedly high. A detailed study of the presented material, the presentation in a lecture and the following discussions, demonstrated several substantial weak points:

- the scientific program is not sufficiently focused on challenging topics in dynamical systems and functional-differential equations,
- most of the research projects and results are of moderate scientific level,
- the connection to real and relevant applications is not strong enough,
- the structure in age and qualification shows deficits, requiring changes.

4. Recommendations

The Commission recommends to the scientific and administrative boards of the Institute and the Academy to consider

a reorientation and a restructuring of Team 2.

We are well aware of the fact that such a measure needs more than just a discussion of the situation of this team, but also of the situation of Mathematics in CAS in general and of the Institute of Mathematics itself. The current scientific program of the Team 2 can be allocated to field Dynamical Systems, which in an up-to date form is missing in the basic mathematical research in CAS. However, there are also more resources required for strong, but inadequately equipped teams, for research areas like Numerical Mathematics or Stochastic and Statistics and new application areas like Life Sciences, Material Sciences or Computational Sciences. Hence, the direction of a reorientation is not a priori obvious.

The decisive factor for the success of all efforts, however, remains attracting a top scientist, who by expertise and vision is able to reorganize the team and to set up a new field of activities at the forefront of research.

The Commission can only stress the demand for changes, but does not suggest specific actions, which might be only suboptimal, since its information on existing side conditions is incomplete.

5. Detailed evaluations

Declaration on the quality of the results and share in their acquisition
This is a relatively small team consisting in total of 9 researchers whose FTE has been more or less constant (approximately equal to 7) during the evaluation period.
The team is focused on a relatively narrow field of research with a common denominator consisting of the investigation of ordinary differential equations and functional differential equations. In particular the following research areas are covered:
1) integration theory and generalized differential equations
2) boundary value problems for ordinary differential equations
3) functional differential equation
4) oscillatory behavior and asymptotic properties of solutions
5) theory of functions.
Topic 1 is focused on the Kurzweil integral and some applications in real analysis. The further potential of this integral e.g. in stochastic analysis could and should have been exploited. A convincing combination of mathematical theory and methods and real life applications is missing.
The scientific achievements of this team cannot be compared with those of the other very strong teams of the Institute of Mathematics. 69% of the delivered outputs were rated in Panel I as "internationally recognized", which does not fit to the high standards of the Mathematics Institute and indicates a deficit in quality.
The only top-level scientist in the team is Jaroslav Kurzweil. His monograph was top rated. However he is 89 years old. Following him with respect to age is Milan Tvrdý (71), whose research

record is respectable. The rest of the team consists of younger members, whose scientific level is not competitive enough for a research institute of CAS. The team suffers still from the loss of the late Štefan Schwabik and obviously misses a person that is setting the necessary standards, determining directions and delivering substantial results.

The publication record of the team is comparatively weak. While the quantity appears reasonable (78 papers), the quality is rather low. The team members publish too often in journals, which have low reputation and are not attracting the attention of the scientific community. Scientists should be selective in choosing journals for publication or in joining their editorial or advisory boards.

The team produced two monographs and two chapters in monographs during the evaluation period. One is a monograph by J. Kurzweil published by World Sci., the other is a graduate text for students of the Palacký University in Olomouc by M. Tvrdý.

Declaration on the involvement of students in research
The members of the team supervised six PhD. students out of which three defended their theses during the evaluation period. One of them later became a member of the team. Some of the PhD. students contributed to the research output of the team.

Declaration on societal relevance
The team has been reasonably involved in undergraduate teaching. Its members participate regularly in the organization and teaching of various conferences, which is appreciated.

Some team members are involved in editorial work for several mathematical journals of either low or moderate quality.

Transfer of mathematical knowledge and methods to solve industrial, economical or social problems does practically not exist. The fact that the team had no substantially funded project, neither in theoretical, nor in applied research, has to be seen very critical.

Declaration on the position in the international and national context
Most of the team members are on a moderate scientific level. A notable exception is, of course, Jaroslav Kurzweil, an internationally top scientist. The members of the team are involved in some international collaboration, however also their partners have to be ranked internationally of moderate quality.

Declaration on the vitality and sustainability
The current composition of the team is far from acceptable and measures have to be taken to improve the personnel resources. The Commission is very worried about the situation of the team as far as the age group is concerned that usually is expected to be the most productive and innovative one. The age structure of the team is distributed between 30-55 with two persons over 65 and a notable lack of young scientists.

Declaration on the strategy and plans for the future
The team is presenting a research plan, which proposes that the members essentially continue in what they have been doing for several years. The lack of necessary changes and innovations is not encouraging. A well thought out scientific strategy and a matching research program are is necessary, though not sufficient for success. External assistance is needed to change the situation of the team. The Commission is aware of the fact that changes cannot be achieved overnight and a fundamental discussion of the strategy for team within the Institute will be necessary. Taking

into account emerging new fields in other institutes of CAS or the Strategy AV21 of CAS, it becomes obvious that basic research in deterministic and stochastic dynamical systems and functional-differential equations is in demand. It should be directly linked to challenging applications e.g. in material sciences, life sciences, environmental sciences, economics and financial sciences, ...

Evaluation of the Team No. 3

Evolution Differential Equations

1. Introduction
The overall topic of this team is the mathematical analysis of model systems mainly arising in continuum dynamics, thermodynamics and reactive flow, diffusion and transport, the development of analytical and finally also numerical methods and tools to derive solutions of the equations, respectively good approximations.

2. Strengths and Opportunities
The team is in the best tradition of Czech Mathematics, in particular in Nonlinear Analysis, Partial Differential Equations, arising as basic equations in Mathematics and as model equations for processes in many areas. As documented by the ratings in Panel I, the team has to be ranked "world leading", a term, which has to be used very carefully not to loose its value. However, the Commission agreed that a substantial part of the results of this team deserve this rating in the strict sense. In particular the progress achieved in analysis for standard and nonstandard equations in fluid dynamics is substantial. Several of the team members can be seen as international leaders in their field. Here we exceptionally want to mention Eduard Feireisl, who succeeded in getting prestigious prizes and financial support for his pioneering research (e.g. ERC Advanced Grant). Most members have perfect international connections and are contributing to Institute´s international exchange program for scientists and students. Also the activity in teaching and education is remarkable.
The team is mainly oriented to basic research and not directly involved in projects with industry and business, however it is supplying mathematical results and tools, necessary to solve challenging problem arising in such projects.
Very important is the involvement of the team in the Nečas Center for Mathematical Modeling, which opens up many perspectives in research and education.

3. Weaknesses and Threats
The fact that too few students are joining the team to work on their theses contradicts the great scientific success of the team and cannot be explained completely.
As all other teams, also this team suffers from the fact that for financial reasons CAS is not attractive enough for the best young scientists.

4. Recommendations
Since the team altogether is running very well, the Commission just encourages it
− to continue and to strengthen the co-operation with Czech and foreign Universities and their teams,
− to pursue with great determination the partnership within the Nečas Center,
− to support the establishment of a research training school for PhD students, with Nečas Center as

possible host,
- to strengthen the links in particular to the team in Stochastics and in Numerical Mathematics within CAS.

5. Detailed evaluations

Declaration on the quality of the results and share in their acquisition
This is a larger team of exceptional quality. It is probably the strongest team evaluated by this Commission.
The main themes of the research program are

1) Mathematical fluid dynamics, covering compressible and incompressible flows, including state dependent viscosities and transport and interactions with solid bodies, in combination with heat conduction and radiation.
2) Mathematical models of solid dynamics, in particular hysteresis, phase field models and phase transitions.
3) Reaction diffusion systems and pattern formation in biological systems.
4) Function spaces

The publication record is outstanding, both with respect to quantity and quality. The team produces first-calls results in all of its areas, which is reflected in the Phase 1 evaluations.
41% of the submitted outputs were rated „*world leading*", 45% „*internationally excellent*", 14 % „*recognized internationally*".

Declaration on the involvement of students in research
Regarding the involvement of students, the team share the difficulties encountered by all CAS institutes. The team has been active in advising master student (4) and doctor students (11). The fact that, despite of the excellence of the scientists, in the evaluation period 4 master thesis were defended, and 0 Ph.D. thesis, indicates that there is a general problem in particular with the PhD studies. Here we refer to the general statements of the Commission, send to the President of CAS.

Declaration on societal relevance
The team's research contributes substantially to the development mathematical results and tools to model, simulate and optimize processes in important application. The team is an important link between basic research in Mathematics and the solution of challenging problem with high impact on the welfare of society. Due to the character of the team, direct involvement with industry and business cannot be expected.
The team is very intensely involved in teaching at all levels, not only in Czech universities, but also at several universities around the world. Members of the team are strongly active for the scientific community at large, such as memberships in advisory boards and editorships in leading international journals.
There have been several activities towards the popularisation of science, notably also through lectures on mathematics for high school students.

Declaration on the position in the international and national context
The team is world leading in most of the areas it is involved with. Several of its members are absolutely outstanding scientists of the high international reputation. The team has numerous international collaborations and has attracted international visitors. Members of the team have had extensive visits abroad at distinguished institutions.

The Departments of Evolution Differential Equations and of Computational Method are intensively co-operating with the teams in Partial Differential Equations, Numerical Methods and Mathematical Modelling and Simulation of the Faculty of Mathematics and Physics (Charles University) and started the Nečas Center for Mathematical Modeling, which has the potential to develop as an important node in an national and international network in the field. The Commission sees several interfaces to teams of CAS closely linked to Mathematics, only a few of which were attributed to Mathematics in the evaluation. Here just one example should be mentioned. The group has started collaboration with the stochastic informatics group of the CAS Institute for Information Theory in the area of stochastic differential equations. This should by all means be continued and possibly strengthened.

The funding situation is very good, in particular also due to the prestigious ERC Advanced Grant for Eduard Feireisl, who is a most prominent figure in both in theory and application oriented Mathematics.

Declaration on the vitality and sustainability
The team is age-wise split into two groups with a large gap between the older and the younger generation. It will be crucial to recruit new excellent member over the next five years to maintain the strength and momentum of the team.
The fact that the team elected an internationally renowned female mathematician as team leader is a visible positive sign for the strengthening the position of female scientists in CAS.

Declaration on the strategy and plans for the future
The group has a convincing research plan covering challenging problems in nonlinear systems of partial differential equations and multi-scale analysis. The program addresses several new directions linking the theoretical research of the team in particular to application in life sciences and engineering sciences. The expertise of this team in the topics 1-3 can be made available for solving challenges in these areas.

Evaluation of the Team No. 4
Numerical Analysis

1. Introduction
This team represents a rather small section of an import field. Numerical Analysis is a highly important sub-discipline of Mathematics, developing and analysing numerical algorithms and

providing tools to compute numerical approximations for solutions to equations and problems, arising in theory or as mathematical model systems in applications. This field was originally started in the Academy by Ivo Babuška, who left in 1968 to US. Babuška became famous for his pioneering work in finite elements, numerical methods of flow through porous media and for incompressible Navier-Stokes-Flow, analytic and numerical multi-scale methods. There are now within CAS two teams focused on Numerical Mathematics:

> this team, studying Finite Element Methods (FEM) and not directly linked,
> the team Computational Methods in the Institute of Computer Science working in Numerical Linear Algebra.

The Team 4 is too small to represent FEM in a broader scale.

2. Strengths and Opportunities

The ratings of the selected outputs of the team, achieved in the evaluation of Panel I, the written report and oral presentation of the team, its program and achievements, gave an overall positive picture. The expertise in theory and methods of Finite Elements and Domain Decomposition is of good quality. It provides a mathematically solid base for developing algorithms and software for numerical simulation in particular of systems of nonlinear partial differential equations, tools for simulation of model equations arising in important applications. Hiring additional young talents in the field of Numerical Analysis, this team could get a catalyst in promoting Numerical Mathematics as theory and application oriented basic research in CAS. In co-operation with the Department for Computational Methods new concepts taking advantage of the multi-scale and parallel structures of advanced computer systems could be developed.

3. Weaknesses and Threats

The age structure and the small size of the team are obstacles for getting to the forefront in the field. As in other teams, resources are missing and the offers, which can be made, are not good enough to convince the best scientists to join the team. The scientific strategy presented mentions new research directions, however is following the individual tracks of the team members to an extend leaving not much space for innovations. The team is in its character method oriented. Putting to much weight on applications bears the risk of reducing the quality of methodical research.

4. Recommendations

Numerical Mathematics deserves to play a stronger role in the basic research of CAS.
The Commission recommends:
- to initiate a cooperation between Team 4 and Department for Computational Methods, with the aim to improve the situation for this discipline,
- to support the Team Numerical Analysis by allocating resources necessary attract highly qualified experts in the field and in particular also young scientist, necessary to balance the age structure.
- to focus the research areas, but also to give more weight to innovation.

5. Detailed evaluations

Declaration on the quality of the results and share in their acquisition
This team is so small that its research could only cover one numerical sub-topic and some selected applications

1. Finite Element Methods (FEM)
2. Applications in particular to model equations in fluid dynamics, solid mechanics, in porous media, to reaction diffusion systems in particular to simulate pattern formation.

The team investigated the following, more traditional subareas:

Bounds for FEM discretisation error, FEM a posteriori error estimates, super-convergence, domain decomposition methods and methods in parallel computing.

33% of the submitted outputs were rated in the evaluation of Panel I as „*internationally excellent*", 67 % as „*recognized internationally*".

The complete list of publications and the quality of the journals confirm the following evaluation: The team is internationally competitive, but not at the leading forefront of numerical research. The standards, set by the founder of Numerical Analysis at IM, Ivo Babuška, are to high to be reached by the team, at least at present.

Declaration on the involvement of students in research

Taking into account the small size of the team, the number of students, advised and integrated in the research, is comparatively good. Due to the general demand for numerical and computational skills and expertise, the field by itself is already attractive. However, also the pedagogical involvement of the team members might be a further factor.

Declaration on societal relevance

Numerical methods, based on mathematical concepts and rigour, belong to the toolbox needed in modern sciences and technology. They are crucial for mathematical modelling, simulation and optimization, keys to master challenges of societal relevance, arising in many areas. Strategy AV21 is going to need the contribution of numerical research in many project areas.

Some members of the team are rather engaged in activities in the scientific community and in popularizing Mathematics and the Sciences.

Declaration on the position in the international and national context

The members of the team are internationally and nationally interlinked with experts in their research area. The team is too small in size, and so far also scientifically not attractive enough to form a strong node in a scientific network. Within the CAS, there is a natural and potential partner, the Department of Computational Methods. However, connections to this scientifically very strong team were not mentioned. The Commission is convinced that the team Numerical Analysis deserves to be developed into a scientific relevant node, maybe in close connection with existing other teams in CAS and the Universities in the Republic.

Declaration on the vitality and sustainability

Size and age structure of the team are obstacles to bringing the Numerical Analysis into the position it deserves. Here, the Institute and the Academy have to act to support this research area, which plays a central role not just in Mathematics, but is not represented enough.

Declaration on the strategy and plans for the future

The research plan has as motto: Traditional topics meet new challenges. As far as the algorithms and the methods side is concerned, the traditional approaches and their continuation seem to dominate. The plan is still more a summary of the interests of the individual members.

New directions like adapting modeling approaches and algorithms to the advanced of computer structures, challenges like high dimensional problems, multi-scale problems, problems with un-

certain data are mentioned, but information, how they will be approached, is missing. The Commission is convinced that focusing of the program is necessary in order to have a chance to achieve any substantial contribution in the new directions. A concentration on the methodological side is recommended, in particular in case of real application projects, where the modeling and the data needs expertise in the application areas which is beyond the scope of a small numerical team. It also does not make sense to cover all mentioned new challenges. The team has to select and to co-operate with colleagues in other teams in CAS or the Universities in Prague or e.g. in Ostrava.

Evaluation of the Team No. 6

Topology and Functional Analysis

1. Introduction

This team is representing two basic disciplines of Mathematics, which have had a long and important tradition in the Academy, like the other areas represented in the Institute of Mathematics. Topology is investigating general objects equipped with geometrical structures, characterizing topological spaces and their relations. Its offspring Functional Analysis is concerned with functions spaces and operators acting on them. Most other mathematical disciplines, in particular analytic disciplines, are relying on topological and functional analytic concepts and tools to a varying extent. During recent years Topology lost territory in the team, whereas Functional Analysis managed to hold position. A main reason for the present situation is that the available conditions are not attractive enough for top scientists in the field.

2. Strengths and Opportunities

The team demonstrated by its research that Banach space theory is the basis for linear and non-linear Analysis. Relevant examples taken from Mathematical Physics are given in its research topic 4, which is concerned with problems of Calculus of Variations, where functional analytic methods play an important role. The contributions in particular to Banach Space Theory and to Operator Theory are excellent achievements. Also the results in complex analysis substantial and were published in very good journals.

The funding through third parties is respectable. Here we just mention the project on "Logic and Topology in Banach Spaces" awarded by FWF (Austria) and Czech Science Foundation for 2016-2018. The research plan for the next period is well designed taking into account properly the high potential of the team.

The team seems to be attractive for the young generation, a fact which very promising for the team and the Institute.

3. Weaknesses and Threats

The range of topics is rather broad and not sufficiently connected, which may create problems to sustain quality and competitiveness, given that the team is not too large in size.

Since Topology has almost vanished, either it should be again represented or - what would be a pity- cancelled in the title.

The age structure of the team is rather unbalanced, a fact which cannot be understood from outside view.

4. Recommendations

The main recommendations of the Committee are
− to fill the gaps concerning the younger generation in the scientific staff,
− to decide on the future position of core topology.
− to explore connections to other mathematical fields represented in CAS, e.g. Numerical Analysis.

5. Detailed evaluations

Declaration on the quality of the results and share in their acquisition
This is a middle-sized team consisting of 14 researchers. Since 2010, it has been constantly losing manpower, and the FTE has been gradually decreasing correspondingly from the initial 12,40 to the current 9,25. The principal losses were those of Petr Honzík and of Václav Zizler, and, very recently, of Miroslav Fiedler.
The team covers several main areas of research:
   1) Banach space theory
   2) Operator theory
   3) Real and complex analysis
   4) Mathematical Physics (thermodynamics, deformable solids)
The main common link is the functional-analytic approach. The team is mostly focused on finding necessary and sufficient conditions for important properties of function spaces and operators acting on them and on applications of such results in various areas of mathematics.
The core areas of Topology are no longer represented in the Department. The intensive efforts to attract a top scientist were not successful so far.
In all the research areas mentioned the team is very strong. The publication record is excellent, both with respect to quantity (161 papers) and quality. The team members regularly publish in highly ranking international academic journals, in vast majority in journals with decent impact factor (134 papers). Although, in their record, the very top journals appear only relatively rarely, many of their papers have been published in journals that are traditionally considered as very good especially in the field of analysis. The team also produced three well-published monographs and two chapters in monographs during the evaluation period.

The evaluation in Panel I rated the sub-mitted output
33% world leading, 62 % internationally excellent and 5% internationally recognized.

Declaration on the involvement of students in research
Despite the well-known obstacles that all institutes of the Czech Academy of Sciences have to face, the team was successful in attracting quite a few young students. The team members were involved in the supervision of 14 PhD. students out of which 7 already defended their thesis during the evaluation period. The team has recently hired two young postdocs. The involved students and postdocs made reasonable contributions to the research record of the team.

Declaration on societal relevance

The team contributes to the development of basic mathematical research with a slight inclination towards applications. The impact to the overall welfare is on a corresponding level. The team is relatively successful as far as the involvement of Ph.D. students is concerned and its involvement in undergraduate teaching is reasonable, as well as in the supervision of bachelor and master theses.

The team members participate regularly in the organization of various conferences, schools and workshops. The team does not cooperate with the business sector, which is adequate given its orientation on the basic research. The team members have been involved in a few activities aimed at the popularization of mathematics and science in general. A notable example of such activity is the series of J. Rákosník's public lectures on digital mathematical libraries.

Declaration on the position in the international and national context
Several members of the team are working at the leading edge of science in their respective fields. The members of the team are involved in many international collaborations with leading scientists and have attracted top quality international visitors. There have been many visits abroad at very distinguished institutions and were often chosen as invited speakers of international conferences. The funding through grants seems to be good and often it involves the young generation.

Date: December 31, 2015

Commission Chair: Professor Willi Jäger